NE-Misc (2) 1952

Thesis/
Reports
Bickford,
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Report On Cooperative Test Of "CONTINUOUS-INVENTORY" FOREST SURVEY SYSTEM In Eastern Maryland

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Northeastern Forest Experiment Station
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July 1952

INTRODUCTION

In the Nation-wide forest survey being made by the Forest Service, U. S. Department of Agriculture, we seek reliable data about present forest areas and timber volumes, and how they change through cutting, mortality, and growth. These data show how well our forests can satisfy present and foreseeable future needs for wood.

Forest-survey procedures have developed gradually during the past 20 years. They embody the experience and study of many able and competent men. But these procedures are still subject to constant scrutiny and criticism in our never-ending search for even more efficient methods.

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In 1947, J. G. Osborne proposed an alternative for the measurement of "drain" and "growth." He called this the "continuous-inventory" system, after Biolley. This proposal was based on measuring change on a series of revisited sample plots; it takes advantage of the reduced sampling error in a series of differences. Osborne, H. W. Camp, R. A. Chapman, and C. A. Bickford developed a plan for testing some elements of this plan in Eastern Maryland and elsewhere. This test was carried out by W. G. Banks, T. J. Grisez, and W. M. Ellsbury under the direction of Camp and A. P. Caporaso.

DESCRIPTION OF THE STUDY

This study was begun in September 1950 in Eastern Maryland with four explicit objectives:

1. To test use of cubic-foot volume classes in photo-interpretation for stratifying forest area.

2. To determine the cost of establishing supplemental ground plots for drain estimation.

3. To determine the relative efficiency of plot versus individual-tree data in growth estimation.

4. To test the possibility of using duplicate aerial-photo coverage of different dates for estimating drain.

Numbers of plots were calculated in the usual manner from formulae adapted from Neyman's "Contribution to the Theory of Sampling Human Population" (Jour. Amer. Statis. Assoc. 33: 101-116, 1938). All photo-plots were examined under a stereoscope and each plot was given a stand-size class according to standard definitions (see Appendix I) and cubic-foot volume class based on the interpreter's estimate (see Appendix II). Ground plots were distributed by volume class, using Neyman's optimum allocation formula; they were randomly drawn from the photo-plots. In addition, for Worcester County, Maryland, all photo-plots in volume classes 1-8 were also given a photo drain class (see Appendix VII).

Half the ground plots in each volume class were designated as "continuous-inventory" plots; they were randomly chosen. The other half of the ground plots were examined, classified, and measured in the standard manner. In addition, time to the nearest minute was recorded from start to finish, but did not include travel time (by automobile).

Each "continuous-inventory" plot was treated as follows, recording time from start to finish of inventory plot and time spent on supplemental plots or strips:

1. Establish plot in regular manner.

2. Increase number of sample trees by including all trees whose radial distance in links from plot center is equal to or less than four times their d.b.h. in inches (4D).

3. Record usual data for trees within 2D; and for trees between 2D and 4D record only species, d.b.h., merchantable height, log-quality data, and tree vigor.

4. Identify all saw-timber trees and smaller sample trees by numbering clockwise from north.

5. Scribe all saw-timber and pole-timber trees, both sample and tally, in such fashion as to permit re-identification of their tree classes within about 10 years.

6. Complete work on the plot in regular manner.

7. Run strip[2] 1 chain wide for 8 chains, marking dead trees and sound and rotten cull trees in a distinctive fashion and noting changes in stand-size class (as estimated on the ground) or in volume class (as seen on the photograph).

Numbers of ground plots and their distribution were estimated for the photo drain test in the usual manner. At each field point, two 1/5-acre plots were examined and all stumps were measured in d.i.b. and height. These plots were completely independent of the inventory growth plots, but this independence is not essential to the method.

In addition, 35 plots established by the Dead River Company in Talmage Town, Maine, in 1939 and 1940 were remeasured in October 1950. These plots provided data on growth, which may be analyzed on a plot or tree basis, as desired.

The field work in the test was completed in July 1951.

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[2] In Cecil and Wicomico Counties, 4 plots of 1/5-acre each were used instead of the strip, but these were so costly they were abandoned in the remaining counties in favor of the strip.

SUMMARY OF RESULTS

USE OF CUBIC-FOOT VOLUME CLASSES

Cubic-foot volume classes were proposed for photo-interpretation with the expectation that cost would be reduced or accuracy would be improved. (Distribution of area by stand-size class would be estimated from proportions of ground plots by these classes in each photo-interpreted volume class. See Appendix III.) The following comparisons were made:

1. Efficiency

Neglecting cost, Snedecor (p. 248, Statistical Methods, 4th Ed. 485 pp., Iowa State College Press, Ames, Iowa., 1948) gives the formula shown in Appendix IV. On this basis, volume classes are 15 percent more efficient for cubic-foot volume estimation than stand-size classes, as shown below:

Variance based on volume class - 1659

Variance based on stand-size class - 1908

$$\frac{1908}{1659} - 1 = 0.15009....$$

These variances were computed from formulae 2 and 3 of Appendix IV.

2. Coefficients of variation (in percent)

Class No.	1	2	3	4	5	6	7	8
Volume	43	57	45	60	36	88	99	128
Stand size	53	48	51	63	67		137	

These coefficients were also computed from formulae of Appendix IV.

3. Cost to obtain equal accuracy

Using 218 ground plots and 2,986 forest photo-plots, an S^2 of 1,659 was obtained from volume-class stratification. To obtain equal accuracy using stand-size class would require 253 ground plots and 2,936 forest photo-plots. At $35 per ground plot and 20 cents per photo-plot, total indicated costs are $8,227.20 using volume class and $9,442.20 using stand-size class. Even greater savings are indicated for Connecticut.

These comparisons have convinced us we should use cubic-foot volume classes in our photo-interpretation. It is probable that with more experience an even better job will be done using volume classes. Stand-size class, on the other hand, has been used 5 years and we cannot expect much further improvement without new and superior techniques.

COST OF SUPPLEMENTAL PLOTS TO OBTAIN DRAIN DATA

Osborne proposed to estimate drain from measurement of change on sample areas. The principal objection raised was the fear of excessive cost. He originally proposed a strip, but because of other objections we first established 4 circular 1/5-acre plots as satellites to the "continuous-inventory" plots. These were so time-consuming that we went back to the supplemental strip.

Detailed comparisons are shown in Appendix VIII. The average time spent on the supplemental strip was 47 minutes for a 2-man crew, but 8 minutes of this (the average of 12 in Appendix VIII is result of more distance) would have been spent on surveying to the plot and returning. Thus, added time averaged 39 minutes per plot versus 91½ for the satellite plots. Osborne based his calculations on 40-chain strips and our sample is one-fifth as large.

In computing cost in dollars, we have to make some assumptions. Our field plots show an average cost of about $35.00 each. About half of this is spent before the car is parked and would not change under any of the alternatives we tested. Total time on the plot averaged 66 minutes. Thus, adding 8 minutes to survey in and walk back would give an average of 74 minutes per plot, or $0.2365... per minute to establish the plot. The supplemental strip would add an average of $9.22 per plot, while satellite plots would add an average of $21.64 per plot. We have no basis for estimating costs of remeasurement.

RELATIVE EFFICIENCY OF TREE AND PLOT DATA IN GROWTH ESTIMATION

Osborne proposed the mapping of all trees on all field plots as a device by which individual trees might be re-identified without resort to marking the trees themselves. This too was regarded as prohibitively expensive.

Use of plot data was proposed as an alternative. The basic comparison rests on 35 remeasured plots in Talmage Town, Maine. The identification of individual trees is most useful in obtaining survivor growth. Ingrowth, cutting, fire, wind, etc., introduce additional problems and occur erratically throughout a forest. These latter factors were omitted in this analysis to obtain a more sensitive test.

A test of significance was made as follows:

$$F = \frac{\frac{(R^2_{y.124} - R^2_{y.12})\Sigma y^2}{1}}{\frac{(1 - R^2_{y.124})\Sigma y^2}{31}} = 18.7$$

in which $R_{y.12}$ is a correlation coefficient based on plot data only and $R_{y.124}$ includes both plot and tree data.

This F is much too large to be random. See Appendix IX for further details.

Numbers of plots required for a sampling error of 10 percent for the State of Maine (equivalent to about 7 percent per 1,000,000,000 cubic feet) were estimated as 242 using plot data only, or 151 if all pole- and saw-timber trees were mapped or otherwise identified.

To complete the picture we need comparative total costs. From our experience in Eastern Maryland, we estimated the cost per plot, identifying individual trees as above, would be about 190 percent of present cost, or $66.50. Assuming 1,000 field plots for Maine would give the following:

Establishment

Plots	1,000 (35)	= $35,000.00
Plots plus trees	849 (35) + 151 (66.50)	= $39,756.50

Remeasurement[3]

Plots	242 (35)	= $ 8,470.00
Plots plus trees	151 (45)	= $ 6,795.00

[3] New inventory needs not taken into account.

Total field

Plots	$43,470.00
Plots plus trees	$46,551.50

These data on cost are admittedly speculative and it is difficult to establish a clear-cut net advantage for either alternative. It is only when we reconsider what we are doing that conclusions can be made. Our present procedure obtains data on about 1½ sample trees per plot. To strengthen our data on quality and distribution of trees by vigor class, we propose to increase our number of sample trees to an average of about 6 per plot. When this is done, the only added step is to record them in order from a fixed point of reference and we will have data on individual trees that will permit us to take advantage of the individual-tree growth data.

These new plots have an average cost of 99 crew-minutes (2-man crew) in comparison with 66 on the old basis (see Appendix VIII). This is a 50-percent increase. Actually, the increase will be somewhat less, since not all the details (included in our test in Eastern Maryland) will be needed. In this test more trees were scribed than is usually needed. Grading all logs in sample trees also contributes to this difference of 33 minutes.

We are convinced that this compromise is more efficient than identifying all or no individual trees.

USE OF DUPLICATE PHOTO COVERAGE TO ESTIMATE DRAIN

This test was proposed in the hope that if we could identify areas cut in the last 10 to 15 years from two sets of aerial photographs (taken 10 to 15 years apart), we could easily afford to examine enough ground plots to meet our accuracy standard. This test has supported that hope, as described below.

A completely satisfying test would have to provide for: (1) Direct measurement of volume removed by cutting; (2) estimation by means of reported production (or consumption) by the various wood-using industries and establishments; and (3) estimation by means of photo drain class. It did not seem feasible to test for all these factors. So our test was aimed at ascertaining if it was feasible to estimate drain from two sets of photographs supplemented by ground sampling.

Using this method, 11-year drain for Worcester County, Maryland, was estimated at 35.855 million cubic feet (164.444 million board feet) with a sampling error of 5.994 million cubic feet, which is equivalent to about 3 percent per 1,000,000,000 cubic feet of drain. This drain was also separated into volume when first photographed and growth since that date and into hardwoods and softwoods by saw-timber and pole-timber trees (see Appendix X). No separation was made of production by commodities or of drain by quality classes.

Forest-survey standards require that commodity drain be separated into logging waste and cutting drain, and broken down into production by commodities. The best way to make this separation would be a special study. But it should be possible to make a separation that is adequate for forest-survey needs by using data that are already available from other surveys or outside sources. The easiest way to do this is to compute from the available data the percentages found in the different elements of commodity drain, and then apply these percentages to our total commodity drain figures.

To obtain data by quality classes will require some supplemental study. The intensity of the study should depend on the accuracy desired.

A cost comparison was also desired. Using photo drain classes, it was found that 125 ground plots and 3,770 photo-plots would meet the National standard of 5 percent per 1,000,000,000 cubic feet for the State of Maryland. This would cost:

$$125\ (14.30) + 3770\ (0.11) = \$2,202.20$$

for the field and photo phases of drain estimation for the State. Actual field costs of the usual method have been:

Production and consumption data	$ 711.72
Woods utilization study	786.36
	$1,498.08

The associated sampling error has not been calculated. The fact that the survey is based on a 100-percent sample of primary products should make the sampling error relatively small. We do not believe, however, that the combination of sampling and reporting error is smaller than the National standard.

These cost data show an apparent advantage for our present procedure, but provide no information on the relative merits of the estimates of drain. Furthermore, it was possible to obtain from the State Forester of Maryland data on production that are not ordinarily available. In the absence of such data, a man-month, more or less, might have been required to obtain comparable data.

Furthermore, it would have been more efficient to combine drain and inventory work rather than do each separately, as was done in the Worcester County test. For example, if all inventory photo-plots (approximately 8,000 forest plots in the State) had been used for drain classification, the number of field plots would be reduced to 110. Doing the photo interpretation as one job, instead of two, would probably cost less than an added 11 cents per photo-plot. If all inventory ground plots are also used as drain plots, a small supplementary sample from heavily cut areas will meet accuracy requirements for drain and will improve accuracy of the inventory. Such a procedure would materially reduce the cost of estimating drain (see Appendix XI).

Length of period between photographs to obtain a useful estimate of drain depends on how fast stumps deteriorate. Stump measurements must be free of bias if a valid estimate is to be obtained. Length of this period probably varies by species, stump size, and climate. It was the feeling of our field crew that 11 years was not too long in this test. This question will be investigated further in Connecticut.

The significant point at this time is that the method is feasible at a reasonable cost. Further experience will be needed to define more clearly when and how to use this method. Connecticut, being a small State with duplicate photo coverage available at a reasonable interval, offers an excellent opportunity to test this method further.

APPLICATION OF RESULTS TO CONNECTICUT

The importance of these results may be better appreciated by considering a specific illustration. The State of Connecticut was chosen because we expect to start photo-interpretation there soon.

The first problem is: How many plots to put on the photographs? We know the total area of Connecticut is 3,205,760 acres and from reappraisal data we estimated the distribution by volume class shown in Appendix XII. Estimated values of $\bar{X}_i$ and S_i were obtained from the Eastern Maryland data, using the last 6 counties

and smoothing by regression. From these data we compute $a = \Sigma P_i S_i$ as $48\frac{1}{2}$ and $b^2 = \Sigma P_i (\bar{X}_i - \bar{X})^2$ as 7,061 for 1/5-acre plots (see Appendix XIII). Experience on forest survey to date shows a cost of about $35.00 per ground plot and 20 cents per photo-plot for the field phase of inventory. Total numbers of ground plots, M_o and of photo-plots, N, are calculated from:

$$M_o = \frac{a(aA + b\sqrt{AB})}{AS^2} \quad \text{and} \quad N = \frac{M_o b^2}{M_o S^2 - a^2}$$

Details of the calculation and required definitions are shown in Appendix XIII, where $M_o = 201$ and $N = 4531$. Number of photo-plots per print is determined by dividing N by number of prints and rounding to the nearest integer. Best results are obtained by completing photo-interpretation and using actual numbers of photo-plots to define P_i (from N_i/N) for distributing M_o by classes. In this illustration we have used P_i from reappraisal data and the formula:

$$M_i = M_o \frac{(P_i S_i)}{(\Sigma P_i S_i)}$$

In some cases it may be preferable to use the approximate P_i, as illustrated, to make most efficient use of personnel or facilities. The loss from using the less precise P_i is not great unless there are gross errors in P_i.

Number of plots photo-interpreted as Class i divided by m_i gives the sampling interval, I_i, for this class. A random number from 1 to I_i is obtained to identify the first plot; every I_ith plot thereafter is also a ground plot. Half the ground plots in each class are "continuous-inventory" plots; these are randomly chosen now. Their extra treatment is described later.

The two-step sampling procedure described by Neyman, on which the above formulae depend, is appropriate for heavily forested areas but its efficiency diminishes as the proportion forested becomes smaller.

Using stand-size classes, the same formulae, and experience values for $\bar{X}_i$ and S_i, we calculated M_o as 395 and N as 8,692, which gives a total field cost of $35 \times 395 + 0.2 \times 8{,}692 = \$15{,}563.40$. Using volume class the cost is $35 \times 201 + 0.2 \times 4{,}531 = \$7{,}941.20$.

The next problem is number of drain plots. These are calculated in an exactly parallel fashion as shown in Appendix XIV.

If only drain is to be estimated, M_o would be 134 and N would be 3,109. But with 4,531 plots marked on the photos for inventory photo-interpretation, it is more efficient to use them also for drain. Then to meet the 5-percent sampling error of 1,000,000,000 cubic feet of drain, S^2 per 1/5-acre plot should be 6.7 cubic feet, and 109 field plots are needed. Their optimum distribution is shown as A - 13; B - 9; C - 16; and D - 71.

The most efficient procedure is to combine the field work by using all ground plots in the estimate of both drain and inventory. Details are shown in Appendix XI. This amounts to making separate calculations of number of plots by drain and by inventory, and then using the larger number. This results in a small reduction in the sampling error of each, but the estimated cost is $7,847 instead of the $8,561 required when the jobs are done separately.

It may be necessary to begin field work before the photo-interpretation is complete. If so, numbers of plots needed for inventory and drain by classes for a county or other subdivision can be estimated, and then adjustments can be made, if needed.

From these data, inventory areas and volumes would be computed as usual. Total periodic drain and its distribution by species and size would be computed following the same general procedure after developing a suitable stump-volume table.

RECOMMENDATIONS AND CONCLUSIONS

This study has convinced us that:

1. Cubic-foot-volume classes are superior to stand-size classes for stratified sampling through use of aerial photographs for estimation of total cubic-foot volume.

2. Cost of supplemental ground plots or strips for estimating drain are too great to warrant attempting a design based on ground plots or strips alone except as a last resort.

3. Use of individual-tree data on growth does effect a significant reduction in unexplained variance, but the increased cost per plot appears to more than offset the smaller number of plots. Possibly more efficient field procedures for tree identification can be devised. But it seems likely that our most efficient procedure is a compromise under which a sample of individual trees would be identified.

4. Use of duplicate photo coverage for estimating drain, especially on a periodic basis, seems to be very promising.

A summary of needed procedures to satisfy requirements of the National manual for the forest survey is sketched below:

Data on inventory volume and area will be obtained as in our present procedure, excepting adjustment shown in Appendix III.

Data on commodity drain will be computed through use of duplicate photo coverage, as explained and illustrated. If such photo-coverage is not obtainable, we must either fall back on the methods that have been used or develop a new design based on ground sampling. For "continuous inventory," supplemental plots will be required if photographs are unobtainable. Supplemental data will be needed to distribute this drain into waste and cutting drain and to distribute production by commodities. This estimate from photographs is equally appropriate under initial survey and re-inventory.

To estimate growth, as defined for the forest survey, we need to know survivor growth, ingrowth, growth on trees cut, growth on trees that die, cull increment, and volume of trees that die.

For initial inventory, no essential changes in present procedure are contemplated. For "continuous-inventory," we propose to obtain all these elements from re-measurements of a series of sample plots as described below:

Survivor growth.--Estimated as explained in Appendix IX.

Ingrowth.--Estimated in a fashion similar to survivor growth, based on plot and tree factors, using measurements of actual ingrowth on remeasured plots.

Growth on trees cut.--Estimated as explained in Appendix X.

Growth on trees that die, cull increment, and volume of trees that die.--Estimated from measured or classified changes on the sample plots.

All these elements of growth may be correlated with volume class and drain class. If so, this relationship should be used to increase the accuracy, or the efficiency, of the forest survey. Dual classification of all photo-plots, showing volume class and drain class, would facilitate use of such a relationship.

On the basis of these results, the forest survey organization in the Northeast has made the following changes in procedure:

1. Photo-interpretation in new units will be made on the basis of cubic-foot-volume classes.

2. One-half the field-inventory plots are being identified as growth plots; on them a larger number of trees will be sample trees for obtaining data on vigor and quality. In addition, sample trees plus saw-timber-tally trees are individually identified by numbering (on the plot record) consecutively from north in a clockwise direction.

3. Use of photo-drain classes will be given a more rigorous test in Connecticut. If results continue favorable, we expect to make it our standard procedure.

The foregoing procedures are dependent on aerial photographs. Should none be available, the first step is to convince someone it is to his advantage to buy them; we would then borrow them for the survey. Should we fail to obtain them in this fashion, some other design may become necessary. Sample coverage by aerial photographs, supplemented by ground plots, is probably the best answer. That problem confronts us in Maine, but so far we have made no decision in the hope we would be able to obtain photographs.

Another possibility is that available photographs may be so old that logging, fire, growth, and so forth in the years since they were made have impaired their usefulness in stratifying forest area. In such a case, if complete new photo coverage is unobtainable, we believe it would be more efficient to supplement the old photographs by flying new sample strips for stratification purposes than to rely solely on ground methods. This new coverage would provide an objective basis for adjusting for change and would permit use of the less expensive photo-interpretation to reduce the amount of costly ground work. This applies equally to an initial inventory, as perhaps in Delaware, or a reinventory of, say, New York when their 1948-49 photographs become too old. In short, if

we cannot induce someone else to buy complete photo coverage, we expect it would be cheaper to buy new sample coverage than to rely on ground methods alone.

APPENDIX

I

STAND-SIZE CLASSES USED

IN PHOTO-INTERPRETATION AND GROUND CLASSIFICATION

1.	Heavy saw timber	10,000 board feet or more per acre
2.	Medium saw timber	5,000-10,000 board feet per acre.
3.	Light saw timber	1,500-5,000 board feet per acre.
4.	Heavy pole timber	Less than 1,500 board feet and more than 600 cubic feet per acre.
5.	Light pole timber	Less than 1,500 board feet and with 200-600 cubic feet per acre.
6.	Seedlings & saplings (well stocked)	Less than 1,500 board feet and less than 200 cubic feet per acre, stocked 40%+.
7.	Seedlings and saplings (poorly stocked)	Less than 1,500 board feet and less than 200 cubic feet per acre, stocked 10-40%.
8.	Unstocked	Less than 1,500 board feet and less than 200 cubic feet per acre, and less than 10% stocked.
9.	Nonproductive	---
0.	Nonforest (This was broken down into 8 standard subclasses)	

II

CUBIC-FOOT VOLUME CLASSES USED IN PHOTO-INTERPRETATION

1. 2,500 or more cubic feet per acre.
2. 1,900-2,500 cubic feet per acre.
3. 1,250-1,900 cubic feet per acre.
4. 600-1,250 cubic feet per acre.
5. 400-600 cubic feet per acre.
6. 200-400 cubic feet per acre.
7. 100-200 cubic feet per acre.
8. 0-100 cubic feet per acre.
9. Nonproductive.
0. Nonforest.

III

DISTRIBUTION OF AREA BY STAND-SIZE CLASS

(Maryland Unit 1)

Volume class	Photo interpretation		Ground-plot data										
	Number of plots	Percent	Item	Stand-size class										
				1	2	3	4	5	6	7	8	9	0	Total
1	272	03.4	No. plots	17	20	16	7	3	--	--	--	--	--	63
			Percent	27	32	25	11	5	--	--	--	--	--	100
2	539	06.7	No. plots	2	8	15	5	2	1	--	--	--	--	33
			Percent	6	24	46	15	6	3	--	--	--	--	100
3	697	08.7	No. plots	--	7	24	7	5	--	--	--	--	--	43
			Percent	--	16	56	16	12	--	--	--	--	--	100
4	543	06.7	No. plots	--	1	20	14	5	3	--	--	--	--	43
			Percent	--	2	46	33	12	7	--	--	--	--	100
5	253	03.1	No. plots	--	--	2	5	4	--	--	--	--	--	11
			Percent	--	--	18	46	36	--	--	--	--	--	100
6	167	02.1	No. plots	--	--	--	--	5	5	--	--	--	--	10
			Percent	--	--	--	--	50	50	--	--	--	--	100
7	138	01.7	No. plots	--	--	1	--	3	5	--	--	--	--	9
			Percent	--	--	11	--	33	56	--	--	--	--	100
8	377	04.7	No. plots	--	--	--	--	1	3	--	1	--	1	6
			Percent	--	--	--	--	17	50	--	17	--	16	100
9	15	00.2	No. plots	--	--	--	--	--	--	--	--	--	--	--
			Percent	--	--	--	--	--	--	--	--	100	--	--
0	5,043	62.7	No. plots	--	--	--	--	--	--	--	--	--	--	--
			Percent	--	--	--	--	--	--	--	--	--	100	--
Total	8,044	--	--	--	--	--	--	--	--	--	--	--	--	--

CALCULATION OF ADJUSTED PROPORTIONS

Volume class	Ground-plot size class									
	1	2	3	4	5	6	7	8	9	0
	Product of volume-class proportion and ground-class proportion									
1	.00918	.01088	.00850	.00374	.00170	--	--	--	--	--
2	.00402	.01608	.03082	.01005	.00402	.00201	--	--	--	--
3	--	.01392	.04872	.01392	.01044	--	--	--	--	--
4	--	.00134	.03082	.02211	.00804	.00469	--	--	--	--
5	--	--	.00558	.01426	.01116	--	--	--	--	--
6	--	--	--	--	.01050	.01050	--	--	--	--
7	--	--	.00187	--	.00561	.00952	--	--	--	--
8	--	--	--	--	.00799	.02350	--	.00799	--	.00752
9	--	--	--	--	--	--	--	--	.00200	--
0	--	--	--	--	--	--	--	--	--	.62700
Adjusted proportion	.01320	.04222	.12631	.06408	.05946	.05022	--	.00799	.00200	.63452
Adjusted percentage	1.32	4.22	12.63	6.41	5.95	5.02	--	.80	.20	63.45
Error ($S_{p_{a_j}}$)	.00348	.00745	.01131	.00946	.01139	.01115	--	.00723	--	.00894
Error as percentage	26.0	17.0	9.0	14.0	19.0	22.0	--	90.0	--	1.4

If P_i = Proportion of photo-interpretation plots in ith volume class

P_{i_j} = Proportion of field plots in ith volume class

K + 1 = Number of volume classes

P_{a_j} = Adjusted proportion of jth volume class

$$P_{a_j} = \sum_{i=o}^{K} P_i P_{i_j}$$

N_i = Number field plots in ith volume class

N = Number points photo-interpreted.

$$S^2_{P_{a_j}} = \sum_{i=o}^{K} P_i^2 \frac{P_{i_j}(1 - P_{i_j})}{N_i} + \frac{1}{N}\left[\sum_{i=o}^{K} P_i P_{i_j}^2 - \left(\sum_{(i=o} P_i P_{i_j}\right)^2\right]$$

IV

FORMULAE

1. Efficiency $$\frac{\frac{1}{S^2_1} - \frac{1}{S^2_2}}{\frac{1}{S^2_2}} = \frac{S^2_2}{S^2_1} - 1 \quad \text{where } S_2 > S_1$$

2. From stratified sampling with optimum allocation

$$S^2_1 = \frac{(\Sigma P_i S_i)^2}{M_o} + \frac{\Sigma P_i (\bar{X}_i - \bar{X})^2}{N}$$

3. Variance from stand-size class data where subsampling was defined by volume class

$$S^2_2 = \frac{(\Sigma P_j S_j)^2}{M_o} + \Sigma P_j (\bar{X}_j - \bar{X})^2$$

$$P_j = \frac{N_j}{N} \quad \text{from photo-interpretation data}$$

$$S^2_j = \frac{\Sigma P_{ij} S^2_{ij} + \Sigma P_{ij} (\bar{X}_{ij} - \bar{X}_j)^2}{\Sigma P_{ij}}$$

$$\bar{X}_j = \frac{\Sigma P_{ij} \bar{X}_{ij}}{\Sigma P_{ij}}$$

$\bar{X}_{ij}$ is mean volume of j^{th} stand-size class of i^{th} volume class.

P_{ij} is proportion of area of j^{th} stand-size class of i^{th} volume class.

S^2_{ij} is volume variance of j^{th} stand-size class of i^{th} volume class.

V

COMPARATIVE COSTS

TO ATTAIN EQUAL ACCURACY FOR INVENTORY VOLUME

(Eastern Maryland)

Given	Volume class	Stand-size class
a	582	627
a^2	338724	393129
b	560	556
b^2	313543	308709
A	35	35
B	0.2	0.2
$\sqrt{AB}$	2.65	2.65
s^2	1659	1659
M_o	218	253
N	2986	2936

Numbers of plots using stand-size class were computed as follows:

$$M_o = \frac{627\left\{35(627) + 2.65(556)\right\}}{35(1659)} = 253$$

$$N = \frac{253\ (308709)}{253\ (1659) - 393129} = 2936$$

Total cost:

Volume class $35(218) + (0.2)\ (2986) = \$8,227.20$

Stand-size class $35(253) + (0.2)\ (2936) = \$9,442.20$

VI

PROPORTION OF FOREST AREA BY VOLUME CLASS AND STAND-SIZE CLASS

(Eastern Maryland; photo-interpretation data only.)

Volume class	Stand-size class						Total
	1	2	3	4	5	6+7+8	
				Per 1,000			
1	44	40	6	0.7	0.3	--	91.0
2	5	103	61	10.0	1.3	0.3	180.6
3	--	26	186	18.0	3.0	1.3	234.3
4	--	1	84	93.0	3.0	0.7	181.7
5	--	--	11	1.7	71.0	1.7	85.4
6	--	--	--	--	52.0	4.0	56.0
7	--	--	--	--	0.3	45.0	45.3
8	--	--	--	--	0.3	126.0	126.3
Total	49	170	348	123.4	131.2	179.0	1,000.0

VII

PHOTO DRAIN CLASSES

A. No apparent cut, probably uncut.

B. No apparent cut, possibly cut.

C. Lightly cut--less than 1/2 crown canopy removed.

D. Heavily cut--more than 1/2 crown canopy removed.

VIII

COST OF SUPPLEMENTAL DATA

(Time in minutes for 2-man crew.)

Stand-size class	Regular plots			"Continuous-inventory" plots		
	Survey to plot[1]	Plot time[2]	Return time[3]	Supplemental area		Plot time[6]
				Strip[4]	Plots[5]	
1	7	88	4	39	82	151
2	10	86	2	46	97	113
3	12	73	3	46	100	105
4	16	60	3	49	106	97
5	6	45	1	50	91	66
6	3	37	1	50	86	55
7	36	64	12	--	--	--
8	2	22	2	52	--	20
9 + 10	7	29	2	--	--	--
Average	10	66	2	47	91	99

[1] Based on 71 plots.

[2] Based on 104 plots.

[3] Based on 67 plots.

[4] Based on strips 1 x 8 chains straddling route to 59 plots.

[5] Based on groups of 4 1/5-acre satellites to 32 plots.

[6] Based on 96 plots.

Average field cost of photo drain plots (1/5-acre)

Class	A	B	C	D
Time in minutes (2-man crew)	$6\frac{1}{2}$	$22\frac{1}{2}$	23	68

IX

SUPPLEMENTAL DATA ON GROWTH BY PLOTS, USING INDIVIDUAL-TREE DATA

All the following data include only trees measured twice:

Y = Growth in the sum of the squares of d.b.h.

X_1 = Sum of squares of d.b.h. at first measurement $\Sigma(dbh)^2$

X_2 = Composition index $\frac{\Sigma(dbh)^2 \text{ fast growing species}}{\Sigma(dbh)^2}$

X_3 = Average d.b.h.

X_4 = Sum of estimated growth of individual trees.

X_4 was obtained from following equations:

		(a)	(b)	r
Spruce	d.b.h. growth =	0.776	+ 0.046 (dbh)	0.1045
Fir	d.b.h. growth =	0.444	+ 0.125 (dbh)	0.2850
Hemlock	d.b.h. growth =	0.544	+ 0.119 (dbh)	0.3826
Pine	d.b.h. growth =	1.019	+ 0.068 (dbh)	0.2671
Cedar	d.b.h. growth =	0.554	+ 0.020 (dbh)	0.0994
Birch	d.b.h. growth =	0.330	+ 0.025 (dbh)	0.1272
Maple	d.b.h. growth =	0.738	+ 0.015 (dbh)	0.0958

Increment in $(dbh)^2 = a^2 + 2a(1+b) \quad (dbh) + b\ (2+b)\ (dbh)^2$

Increment in $\Sigma(dbh)^2 = Na^2 + 2a(1+b)\ \Sigma(dbh) + b\ (2+b) \quad (dbh)^2$

r_{y1}: 0.7289 $\qquad R^2_{y.12} = 0.57776...$

r_{y2}: 0.3295 $\qquad R^2_{y.124} = 0.73666...$

r_{y3}: 0.1892* $\qquad s^2_{y.12} = 2113057$

r_{y4}: 0.8454 $\qquad s^2_{y.124} = 1317854$

---

* Not significantly different from zero; used no more.

For 10-percent accuracy:

$$N_{12} = \frac{s^2_{y.12}}{(93.374)^2} = 242$$

$$N_{124} = \frac{s^2_{y.124}}{(93...)^2} = 151$$

X_4, the sum of estimated growth of individual trees, was obtained by using all the mapped trees on each of the 35 plots. The above equations of d.b.h. growth over d.b.h. by species were fitted to the data. These equations were then used to estimate growth in the sum of squares of d.b.h. The sum of these estimates over all species is the X_4 used in this analysis.

If such a procedure were used by forest survey, growth would be expressed in volume, both cubic feet and board feet, and appropriate changes would be made in the other variables. The equations or curves by species would be based on sample trees. They might be strengthened by separate preparation by vigor class, which reduces unexplained variation in growth of individual trees.

X

SUPPLEMENTAL DATA ON PHOTO DRAIN ESTIMATION

(Worcester County, Md.)

Separation of drain[1] into initial volume and growth

Photo drain class	Commodity drain		Initial volume when photographed		Growth since photographing	
	Cu.ft.	Bd.ft.	Cu.ft.	Bd.ft.	Cu.ft.	Bd.ft.
A	1,304,603	--	999,631	--	304,972	--
B	944,389	4,875,675	675,819	3,244,175	268,570	1,631,500
C	8,000,606	50,935,412	6,525,612	40,022,539	1,474,994	10,912,873
D	25,605,113	108,633,393	21,289,467	85,651,977	4,315,646	22,981,416
Total	35,854,711	164,444,480	29,490,529	128,918,691	6,364,182	35,525,789

[1] These data may be distributed by classes of growing stock shown in the following table; they may also be separated by species, if needed, insofar as species may be identified from a stump.

The above separation was made from:

$$\frac{\text{Volume when cut}}{1 + nr} = \text{initial volume}$$

Growth of cut trees = volume when cut minus initial volume.

n = number of years from initial photograph to cutting.

r = growth percent of surviving trees (exclude mortality and ingrowth).

Distribution of estimated 11-year commodity drain

for Worcester County, Md.

Photo drain class	Class of materials	Item	Cubic feet (in thousands)	Board feet (in thousands)
A	Hardwoods	Pole timber	1,305	--
B	Softwoods	Pole timber	14	--
		Saw timber	930	4,876
C	Hardwoods	Saw timber	4,227	30,540
	Softwoods	Saw timber	3,773	20,395
D	Hardwoods	Pole timber	1,103	--
		Saw timber	4,401	27,510
	Softwoods	Pole timber	3,036	--
		Saw timber	17,065	81,124
All hardwoods		Pole timber	2,407	--
		Saw timber	8,628	58,050
		Total	11,035	58,050
All softwoods		Pole timber	3,051	--
		Saw timber	21,769	106,394
		Total	24,819	106,394
Total		Pole timber	5,458	--
		Saw timber	30,396	164,444
		Total	35,855	164,444

XI

DISTRIBUTION OF GROUND PLOTS FOR CONNECTICUT, BY VOLUME AND DRAIN CLASS

Volume class	Estimated proportion* of total area per 1,000 for drain class—					Estimated number of photo plots	Number of ground plots required for inventory alone	Number of ground plots by drain class--					Recommended total number of ground plots for both inventory and drain
	A	B	C	D	Total			A	B	C		D	
								M_{ij}	M_{ij}	M_{ij}	N_{ij}	N_{ij}	
1	4	1	1	—	6	27	5	3	1	1	—	—	5
2	12	3	3	—	18	82	11	7	2	2	1	—	11
3	37	4	2	—	43	195	21	18	2	1	1	—	21
4	65	4	2	1	72	326	28	25	2	1	1	1	29
5	156	4	2	3	165	748	54	51	1	1	1	4	57
6	87	4	2	5	98	444	29	26	1	1	1	7	35
7	84	6	10	20	120	544	31	22	2	3	4	26	54
8	35	4	14	25	78	353	20	8	1	4	6	33	48
Subtotal	480	30	36	54	600	2,719	—	160	12	14	15	71	260
Proportion of forest land	800	50	60	90	1,000	—	—	—	—	—	—	—	—
9	—	—	—	—	2	9	—	—	—	—	—	—	—
0	—	—	—	—	398	1,803	2	2**	—	—	—	—	20**
Total P	—	—	—	—	1,000	—	—	—	—			—	
N						4,531	201	162 (180)	12		17	71	280
N_j	13	9	16	71		109	—	—	—		—	—	—

$$M_{ij} = M_i \frac{(P_{ij})}{(P_i)}$$

$$N_{ij} = N_j \frac{(P_{ij})}{(P_j)}$$

Larger number (of these two) is used for number of ground plots by subclass. Direct comparison is possible only for Drain Class C. In other cases it was obvious which number would be larger.

* These proportions are estimated without photo-interpretation data; in application, this distribution should be made when photo-interpretation data are complete.

** Calculated M for this class is 2 but recommended number is 20 to minimize influence of photo-interpreting poorly stocked forest land as nonforest.

XII

ESTIMATED PROPORTIONS OF LAND AREA OF CONNECTICUT, BY CLASSES[1]

Volume class	Proportion of total land area (per thousand)	Drain class	Proportion of forest area (percent)
1	6	A	80
2	18	B	5
3	43	C	6
4	72	D	9
5	165	Total	100
6	98		
7	120		
8	78		
9	2		
0	398		
Total	1,000		

[1] Estimated from reappraisal data.

XIII

CALCULATION OF NUMBERS OF PLOTS FOR INVENTORY

(Connecticut)

Volume class	P_i (per thousand)	$\bar{X}_i$	S_i	P_iS_i	$\frac{P_iS_i}{\Sigma P_iS_i}$ (per thousand)	M_i
1	6	425	182	1.1	23	5
2	18	324	145	2.6	54	11
3	43	255	121	5.2	107	21
4	72	184	95	6.8	140	28
5	165	138	79	13.0	268	54
6	98	116	71	7.0	144	29
7	120	94	63	7.6	157	31
8	78	89	61	4.8	99	20
9	2	2	2	--	--	--
0	398	1	1	0.4	8	2
Σ	1,000	--	--	48.5	1,000	201

Ground plot cost: (A) = $35

Photo plot cost: (B) = 0.2

$\Sigma P_iS_i = a = 48.5$

$\Sigma P_i(\bar{X}_i - \bar{X})^2 = b^2 = 7061$

$a^2 = 2352$

$b = 84$

$\sqrt{AB} = 2.65$

$\bar{X} = \Sigma P_iX_i = 85$; $\Sigma P_i(\bar{X}_i - \bar{X})^2 = 7,061$

Estimated total volume is 3205760 x 5 x 85 = 1,362,448,000 cubic feet.

Whence (allowable error)$^2 = S^2 = \left(\frac{5}{100}\right)^2 \frac{(1,000,000,000)}{(1,362,448,000)} 85^2 = 13.26$

$$M_o = 48.5 \frac{((48.5)(35) + (84)(2.65))}{35(13.26)} = 200.6 \text{ or } 201$$

$$N = \frac{201(7061)}{201(13.26)-2352} = 4530.6 \text{ or } 4531$$

XIV

CALCULATION OF NUMBERS OF PLOTS FOR DRAIN ESTIMATION

Drain class	P_i (Percent)	$\bar{X}_i$	S_i	P_iS_i	$\frac{P_iS_i}{P_iS_i}$ (Percent)	M_i	M_i
A	80	2	4	320	116	12.6	13
B	5	40	45	225	81	8.8	9
C	6	90	70	420	152	16.6	16
D	9	185	200	1800	651	71.0	71
Σ	100	--	--	2765	1000	109.0	109

$$M_o = \frac{27.65\ (\ 27.65(15) + 55(1.28)\ )}{15(6.6677)} = 134$$

$$N = \frac{134(2992)}{134(6.6677) - 764.5225} = 3109$$

But N for inventory is 4531 and to use same photo plots

$$S^2 = \frac{a^2}{M_o} + \frac{b^2}{N} \text{ or } 6.6677 = \frac{764.5225}{M_o} + \frac{2992}{4531}$$

and $M_o = 109.1\ldots$

Agriculture-Forest Service-Upper Darby, Pa.